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Mass Symmetry

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ABSTRACT

There is symmetry in the nature. Then, there should also be symmetry in physics since physics describes the phenomena of nature. In fact, it occurs in most of the phenomena explained by physics as for example: a particle has positive or negative charges, spins up or down, north or south magnetic poles. In this form, the particle should also have mass symmetry. For convenience and due to later explanations, I call this mass symmetry or mass duality as follows: mass and mass cloud. The mass symmetry can be corroborated in the experiments of the hydrogen spectrum, the Bohr model and the solution of the Schrödinger equation. The mass cloud is located in the respective orbitals given by the Schrödinger equation. The orbitals represent the possible locations or places of the particle which is determined probabilistically by the respective Schröndiger equation.

For the proton, part of the mass of the uncharged proton is distributed in the orbital or mass cloud around the mass that contains the positive charge. Thus, the positive charge in the proton is concentrated in its mass nucleus with an uncharged mass cloud around its nucleus distributed in the orbitals. For the electron, part of the mass of the uncharged electron is distributed in the orbital or mass cloud around the mass that contains the negative charge. Thus, the negative charge in the electron is concentrated in its mass nucleus with an uncharged mass cloud around its nucleus distributed in the orbitals.

For example, in the formation of the hydrogen atom, a part of the mass cloud of the proton interacts with the mass cloud of the electron, and the total mass energy lost in this interaction is transformed into electromagnetic energy according to Einstein's equation: $E=mc^2$ and the variant mass formula discovered and developed by myself. Then, the two particles join together due to this interaction and the electrostatic force between the two particles. Therefore, the electron and proton are bound together in the hydrogen atom by the mass cloud of the electron and proton with some mass cloud lost in the interaction and converted to electromagnetic energy or photons. Then, it is right this mass symmetry, since the electron and the proton in the interaction of the mass cloud lose mass but do not lose electric charge. In this form, it is justified the existence of a mass cloud. In the formation of the Hydrogen atom, the electron-proton system when approaching gains a potential energy of 27.2 eV (13.6 eV*2) but then when the electron bond occurs in the shell with quantum state n =1, energy of 13.6 eV is emitted as electromagnetic energy or photons and the remaining 13.6 eV remains as kinetic energy of the electron. Then, the Hydrogen atom has 13.6 eV of additional energy/mass than the sum of the energy/mass of the proton plus the electron. Therefore, 13.6 eV is needed to ionize the Hydrogen atom and expel the electron from the atom. The mass/energy reduction of the proton and electron is 13.6/2 eV for each particle due the emission of 13.6 eV as electromagnetic energy.

Therefore, the main function of the mass cloud is the binding energy. The mass cloud interaction generates binding energy between the electrons and the nucleus in the atom through the protons and between the nucleons in the nucleus: protons with protons, neutrons with neutrons, and protons with neutrons. The nuclear force between two nucleons is characterized by being strong and short-range. Also, it can be justified by the existence of the mass cloud: the mass clouds of nucleons within the nucleus interact with each other without any effect on the proton charge.

This scientific research presents evidence of the existence of the mass symmetry based in the Einstein's equation and in the Variant Mass formula for the Electron in the atom discovered and demonstrated by myself where experimental results are detailed.

Keywords: *Mass symmetry, Mass/Energy Einstein equation, Giovanni Alcocer Variant Mass fundament theory/formula, Hydrogen atom, Radial probability density, Schröndiger equation, Bohr model, Muonic atom, Ionized helium atom, Helium nucleus, Nucleons, Antiparticles, Proton Antiproton, Muon Antimuon, Neutral Pion and Neutral Antipion, Hydrogen molecule H_2, Ionized hydrogen molecule H_2^+, Oxygen molecule O_2.*

1. Evidence of Particle Symmetry and Postulates of Mass Symmetry

In this scientific article, the mass symmetry is researched, demonstrated and explained with some known particles: electrons, protons, neutrons, pions, muons, nucleons, particle and antiparticles, atomic bonds and diatomic molecules.

In nature, the phenomena and interactions between particles follow a symmetrical pattern: positive charge and negative charge, south and north poles of the magnetic field, the spin up and down of electrons in atoms, and the spins of nucleons in the nucleus, which are obvious examples of the existence of these symmetries in nature [7],[8],[9]. Therefore, it seems justifiable the existence of mass symmetry [6].

The existence of mass symmetry: mass and mass cloud is supported by two postulates:

(1) The value of the mass cloud of a particle is equal to the value of its mass. Thus, if "m" is denoted as the positive mass of a particle, the mass cloud m* of this same particle also has mass. Then, m*=m for the same particle. In this form, the direction of the gravitational force for the positive mass and the mass cloud is the same.

(2) The mass of a particle cannot interact with the mass cloud of the same particle, neither partially nor totally. However, the interaction occurs between the mass cloud of one particle and the mass cloud of another particle, either partially or totally.

Then, the loss of mass cloud due the interaction is converted into electromagnetic energy or photons based on Einstein's equation and the variant mass formula [1]-[5]. Thus, as a result of this interaction, electromagnetic radiation or photons will be emitted:

$mc^2+m^*c^2 \rightarrow E_\gamma + E_\gamma$.

In this process two or rarely three photons will be emitted. It is because two photons are needed at least to hold conservation of momentum. The loss of mass cloud and converted in electromagnetic energy is given by the variant mass formula discovered and developed by myself in the article [1]: "The Fundament of the Mass: The Variant Mass of the Electron at the atom. Experimental results: Ionization energy of the electrons at the atom. Bound of the Diatomic Molecules. From Bohr to Schröndiger. Deduction of the radius formula with the analogy of the atom with the blackbody research from Planck and harmonic oscillators. Why the electrons orbit around the nucleus with specific radius for the case of electron as particle. Why the electron doesn´t radiate energy at the stationary levels as a wave and as a particle. Diffraction and Interference for the electron by using the Fourier Approach and Wave Properties".

Besides, the emission of the electromagnetic radiation occurs for accelerated charged particles. Maxwell's theory showed that electromagnetic waves are radiated when charges particle accelerate [2],[10]. The electromagnetic radiation emitted is obtained with the formula of the variant mass of an accelerated charged particle developed by myself [2].

For other hand, there is emission of gravitational energy for a particle orbiting a large object and for a binary star. The formula that describe the mass of a particle which emits gravitational energy is obtained and demonstrated by myself in the article [3]: The Fundament of the Mass and Effects of the Gravitation on a Particle and Light in the mass, time, distance, velocity, frequency, wavelength: Variant Mass for a Particle which emits Gravitational Energy for a particle orbiting a large Planet or Sun and for a Binary Star and Variant Frequency for the Light passing close a Gravitational Field from a Massive Object (Sun).

2. Hydrogen Atom

The Hydrogen atom consists of one proton and one electron. The value of the particle mass is equal to the value of its mass cloud as it was established in the first postulate. Thus, the sum of the mass and the mass cloud of the proton is 938.27 MeV/c^2 and the sum of the mass and the mass cloud of the electron is 0.511 MeV/c^2.

According to the Bohr model for the hydrogen atom [1], there are allowed orbits for the electron in which the angular momentum is $\frac{nh}{2\pi}$ where n is the principal quantum number: 1, 2, 3…. The theoretical results of the binding energy of the electron and the probabilistic distance r of the electron location are compatible with the Bohr model and spectroscopy experiments and with the solution of the Schrödinger equation in quantum mechanics which gives the maximum probability density versus r for the hydrogen atom [6].

According to the Bohr model and for the hydrogen atom and depending on the quantum number n, the velocity (v), energy (E) and radius (r) of the electron are [1]:

$$v = \frac{Ze^2}{2\varepsilon_o nh} \qquad E = \frac{-z^2 e^4 m_o}{8{\varepsilon_o}^2 h^2 n^2} \qquad E = \frac{-13.6z^2}{n^2}$$

$$r = \frac{n^2 h^2 \varepsilon_o}{\pi m_o Ze^2} \qquad a_o = \frac{\varepsilon_o h^2}{\pi m_o e^2} \qquad r = \frac{n^2 a_o}{Z}$$

Z: atomic number of the atom

e: charge of the electron

ε_o: vacuum permittivity

n: main quantum number, electron energy level, orbit of the electron

h: Planck constant

m_o: rest mass of the electron

Values of constants:

Z=1 (Hydrogen atom)

$h=6.63*10^{-34}$ J-s

$\varepsilon_o=8.85*10^{-12}$ Farad/m

$\pi=3.1416$

$m_o=9.11*10^{-31}$ kg

$e=1.602*10^{-19}$ C

$r_o=0.529$ $\dot{A}$ Bohr radius, radius for the first level of the Hydrogen atom

1 $\dot{A}= 10^{-10}$m   1nm$=10^{-9}$ m

For Z=1, it is obtained from the formula of E and r:

$$(E)(r)= \left(\frac{-z^2 e^4 m_o}{8{\varepsilon_o}^2 h^2 n^2}\right) \left(\frac{n^2 h^2 \varepsilon_o}{\pi m_o Ze^2}\right)$$

$$=\frac{-Ze^2}{8\pi\varepsilon_o} \qquad Z=1$$

$$=\frac{-e^2}{8\pi\varepsilon_o}$$

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(E)(r)=0.7194 eV-nm

$$\mathrm{E(eV)} = \frac{0.7194}{\mathrm{r(nm)}}$$

For each quantum number n, the velocity (m/s), the radius (m), and the energy (eV) of the electron in the respective shell or quantum level are [1]:

Table 1. Values of velocity (v: m/s), radius (r: m), and energy (E: eV) for each quantum number n

n	v (m/s)	r (m)	E=hf Ecinetica (ionization energy) (eV)
1	2186946,852	5,29636E-11	-13,61585307
2	1093473,426	2,11854E-10	-3,403963266
3	728982,2839	4,76672E-10	-1,512872563
4	546736,7129	8,47417E-10	-0,850990817
5	437389,3704	1,32409E-09	-0,544634123
6	364491,142	1,90669E-09	-0,378218141
7	312420,9788	2,59522E-09	-0,277874552

The graph of E (eV) versus r (nm) is shown by using the formula: E(eV)=0.7194/r (nm) and the data in the table (1) shown:

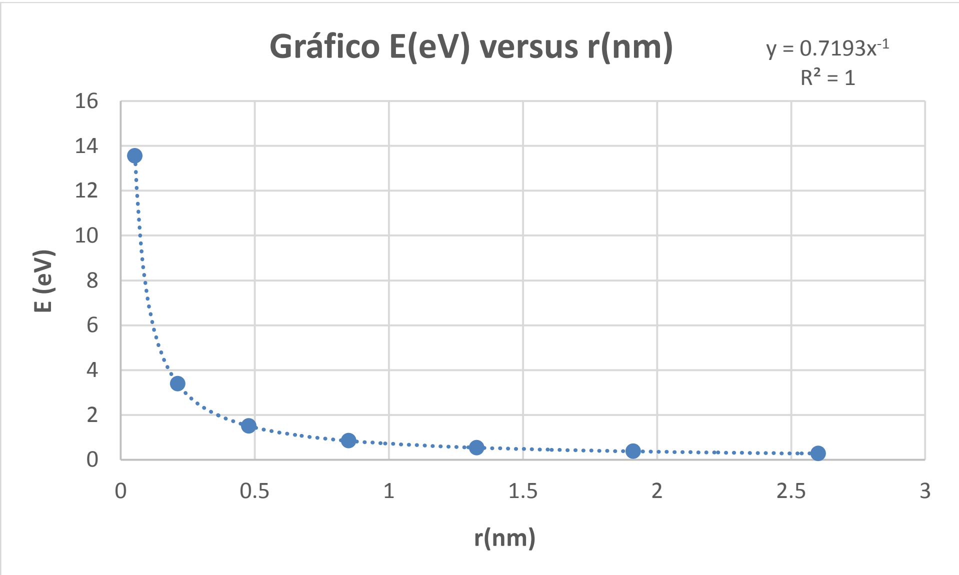

Fig.1. Energy Variation (eV) versus r(nm) for the different quantum levels n based on the Bohr model

This diagram is also obtained with data obtained through spectroscopy experiments performed on the hydrogen atom. The spectroscopy results are compatible with the Bohr model represented in the previous graph and the Schrödinger equation which gives the maximum value of the probability density of finding electrons as a function of r for the different shells or quantum levels of the hydrogen atom [6].

In order to test the development formula of the variant mass for the ionization emission energy of the electron for the Hydrogen atom, some calculation by using Quantum Mechanics is done. Then, the mass results for both methods are compared [1].

First energy level Hydrogen atom: -13.6 eV n=1

Second energy level Hydrogen atom: -3.4 eV n=2

If the electron jumps from the second level to the first level, the energy emitted is (13.6-3.4) eV=10.2 eV and the mass of the electron must lose this equivalent mass-energy. Thus, the lost mass of the electron which is equivalent to the mass-energy of the electromagnetic radiation emitted occurs during the transition from the second level to the first level. In mathematical formulation, it is as follows: $(m_o-m)c^2=hf=K$ where m_o is the electron mass before the transition and m is the electron mass after the transition, K is the kinetic energy of the electron and E=hf is the energy of the photon emitted (electromagnetic radiation) [1].

If the electron at the first level (n=1) (quantum level 1s) leaves from the atom, it is necessary to add an energy of 13.6 eV. Also, it corresponds to the energy emission of the electron in order to bring the electron from the infinite to the first level. Therefore, the energy emission of the electron is: $hf= m_oc^2- mc^2=13.6$ eV where m_o is the electron mass before the transition and m is the electron mass after the transition [1].

It is in coincidence with the development formula for the energy emission of the electron at the atom: variant mass of the electron at the atom [1]. The mass electron calculation with the mass development formula is as follows:

$$m = m_o e^{-\left(\frac{v^2}{2c^2}\right)} \qquad m_o=511797.7528 \text{ eV}/c^2$$

The velocity is given by this formula:

$$v = \frac{Ze^2}{2\varepsilon_o nh}$$

It is interesting to mention that this formula doesn't include the mass of the particle. Therefore, the orbits have specific values for the particle independent of the mass of it. By replacing the values for Z (Z=1), e, ε_o, n (n=1), h, it is obtained:

v=2186946.852 m/s. By replacing this value at the mass formula, it is achieved:

$mc^2=511784.1541$ eV

$m_oc^2=511797.7528$ eV (energy in eV for the electron mass: $9.11*10^{-31}$ kg)

Then, it is obtained:

$hf= m_oc^2- mc^2$

hf=511797.7528-511784.1541

hf=13.59 eV. It is the same value that the last calculation by using quantum mechanics for the energy emission of the electron.

It is possible to do the same for the second level of the Hydrogen atom. The ionizing energy for the electron at the second level is: -3.4 eV. Then, if the electron at the second level (n=2) leaves from the atom, it is necessary to add an energy of 3.4 eV. Also, it corresponds to the energy emission of the electron in order to bring the electron from

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the infinite to the second level. Therefore, the energy emission of the electron is: hf= m_oc^2- mc^2=3.4 eV where m_o is the electron mass before the transition and m is the electron mass after the transition [1].

It is in coincidence with the development formula for the energy emission of the electron at the atom: variant mass of the electron at the atom [1]. The mass electron calculation with the mass development formula is as follows:

$m = m_o e^{-(\frac{v^2}{2c^2})}$ m_o=511797.7528 eV/c^2

The velocity is given by this formula:

$$v = \frac{Ze^2}{2\varepsilon_o nh}$$

By replacing the values for Z (Z=1), e, ε_o, n (n=2), h, it is obtained:

v=1093473.426m/s

By replacing this value at the mass formula, it is achieved:

mc^2=511794.3531 eV

m_oc^2=511797.7528 eV (energy in eV for the electron mass: 9.11*10^{-31} kg)

Then, it is obtained:

hf= m_oc^2- mc^2

hf=511797.7528-511794.3531

hf=3.39 eV. It is the same value that the last calculation by using quantum mechanics for the energy emission of the electron.

It is showed at the next table (2) the values of the velocities (m/s), radius (m), energy of the ionization (eV) for the different levels of energy of the hydrogen atom. Also, it is showed the mass of the electron m (eV/c^2) after the emission of the electromagnetic radiation by using quantum mechanics (mc^2=m_oc^2-hf) and with the formula of the variant mass for the electron at the atom after the energy emission [1]: $mc^2 = m_oc^2e^{-(\frac{v^2}{2c^2})}$.

Table 2. Values of the velocities (v m/s), radius (r m), ionization energy (E eV), mass of the electron m (eV/c^2) after the emission of the electromagnetic radiation, for different quantum levels of energy (n) of hydrogen atom

n	v	r	E=hf (eV) Ecinetica (ionization energy)	mc^2=m_oc^2-hf	$mc^2 = m_oc^2e^{-(\frac{v^2}{2c^2})}$
1	2186946,852	5,29636E-11	-13,61585307	511784,137	511784,1541
2	1093473,426	2,11854E-10	-3,403963266	511794,3488	511794,3531
3	728982,2839	4,76672E-10	-1,512872563	511796,2399	511796,2418
4	546736,7129	8,47417E-10	-0,850990817	511796,9018	511796,9029
5	437389,3704	1,32409E-09	-0,544634123	511797,2082	511797,2089
6	364491,142	1,90669E-09	-0,378218141	511797,3746	511797,3751
7	312420,9788	2,59522E-09	-0,277874552	511797,4749	511797,4753

The accuracy of the formula is demonstrated theoretically. Besides, the table (2) shows that when the velocity decreases (at the different levels of energy of the Hydrogen atom) the mass increases because when the velocity

decreases, the distance r of the electron increases and there is less emission of electromagnetic energy for the electron to bind in the respective shell from the infinite or it is needed less emission of energy to eject the electron from the atom. Besides, levels which are closest to the nucleus (less distance r to the nucleus) have higher velocities than the farthest because it is needed more velocity and kinetic energy ($K = \frac{1}{2}mv^2 = m_oc^2 - mc^2$=hf=energy of the photon emitted) so that the electron does not fall into the nucleus. Nevertheless, the potential energy ($V = -k\frac{e^2}{r}$) decreases (more negative) when the distance r decreases [1]. Then, it is necessary to add more energy to eject the electron from this shell since it is necessary to overcome a greater potential energy (more negative).

V decreases (more negative) when r decreases. If we want to reduce the distance between the electron m and the nucleus M by doing a transition from one orbit to another orbit, it is necessary that the electron m emits energy (emission as electromagnetic energy or photons) at this transition. The emission of this energy produces a decrease of the electrical potential energy (more negative) and thus, it is necessary to add more energy to eject the electron from this shell since it is necessary to overcome a greater potential energy (more negative). It produces also an increase of the kinetic energy and the electron has more velocity [1].

V increases (less negative) when r increases. If we want to move the electron m from their respective orbit to another orbit increasing its distance from the nucleus, it is necessary to apply an external force or to gives an additional energy to the system. The work done by this force produces an increase of the electrical potential energy (less negative). Thus, it is necessary to add less energy to eject the electron from this shell since it is necessary to overcome a lower potential energy. Part of the work done by this force or the additional energy given produces a decrease of the kinetic energy and the electron has less velocity. Nevertheless, the electron has restricted positions or radius to do the transitions from one orbit to another until it gets the stationary orbit with stationary energy level. For this reason the velocity of the respective shell doesn´t depends of the mass of the particle or the radius of the shell: $v = \frac{Ze^2}{2\varepsilon_o nh}$. Besides, at these stationary orbits or states, the electron doesn´t emit electromagnetic energy or photons. The electron only does the emission of the electromagnetic energy or photons at the transition from one orbit to another [1].

3. Radial Probability Density of the Electron

It is necessary to examine the results of the solution of the Schrödinger equation to obtain the distribution of the mass cloud around the proton.

The radial probability density of the electron in the hydrogen atom is: $P(r) = r^2|R(r)|^2$. The maximum probability occurs when the probability densities have spherical symmetry where $r_{max}=n^2a_o$ and for l=n-1. Therefore, there is a spherical distribution at n=1 l=0: $r_{max}=a_o$, n=2 l=1: $r_{max}=4a_o$, n=3 l=2 $r_{max}=9a_o$, n=4 l=3: $r_{max}=16a_o$. In addition to the states for which the probability densities have spherical symmetry, there are states whose probability density distribution is not spherical [6].

The radial wave function R(r) is shown below for some states:

$R(r) = \frac{2}{(a_o)^{3/2}} e^{-\frac{r}{a_o}}$ n=1 l=0 state 1s

$R(r) = \frac{2}{(2a_o)^{\frac{3}{2}}} (2 - \frac{r}{a_o}) e^{-\frac{r}{2a_o}}$ n=2 l=0 state 2s

$R(r) = \frac{1}{\sqrt{3}\,(2a_o)^{3/2}} \frac{r}{a_o} e^{-\frac{r}{2a_o}}$ n=2 l=1 state 2p

$R(r) = \frac{4}{27\sqrt{10}\,(3a_o)^{3/2}} (\frac{r}{a_o})^2 e^{-\frac{r}{3a_o}}$ n=3 l=2 state 3d

$R(r) = \frac{1}{\sqrt{322560}\,(a_o)^{3/2}} (\frac{r}{2a_o})^3 e^{-\frac{r}{4a_o}}$ n=4 l=3 state 4f

The radial wave function R(r) versus (r/a_o) is shown in the next figure 2 for n=1 l=0 state 1s (r and a_o in A):

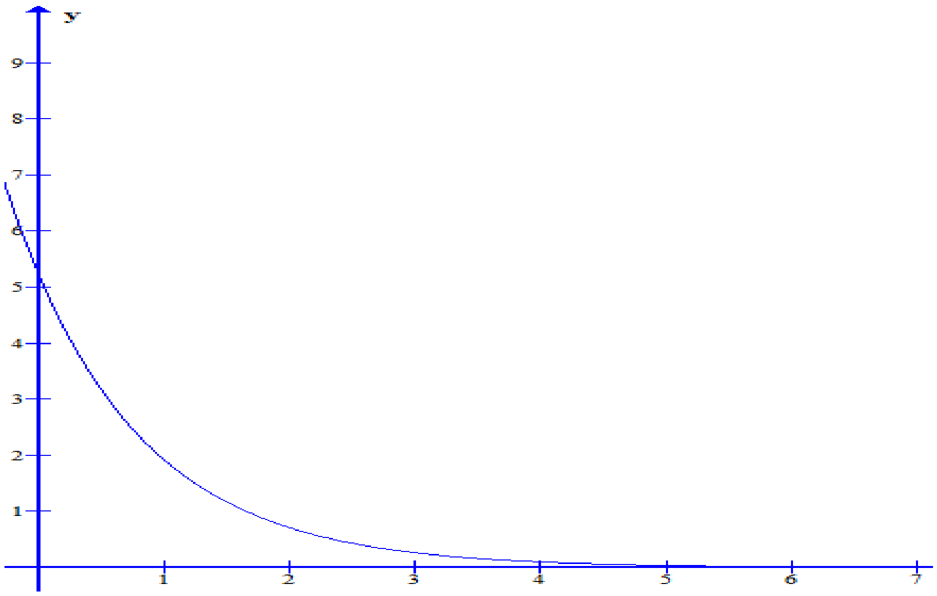

Fig.2. Radial wave function R(r) versus r/a_o (n=1 l=0, r and a_o in A)

It is possible to demonstrate that the most probable distance of an electron in the state n=1 l=0 is a_o.

$P(r) = r^2|R(r)|^2 = r^2 \frac{4}{{a_o}^3} e^{-\frac{2r}{a_o}}$

$\frac{dP(r)}{dr} = \frac{4}{{a_o}^3} (-\frac{2r^2}{a_o} + 2r) e^{-\frac{2r}{a_o}}$

$r=a_o$

Besides, it is possible to demonstrate that the most probable distance of an electron in the state n=2 l=1 is $4a_o$.

$P(r) = r^2|R(r)|^2 = \frac{r^4}{24{a_o}^5} e^{-\frac{r}{a_o}}$

$\frac{dP(r)}{dr} = \frac{1}{24{a_o}^5} (-\frac{r^4}{a_o} + 4r^3) e^{-\frac{2r}{a_o}}$

$r=4a_o$

In the same form, it is possible to demonstrate that the most probable distance of an electron in the state n=3 l=2 is $9a_o$ and in the state n=4 l=3 is $16a_o$. Thus, the maximum probability occurs when the probability densities have spherical symmetry where $r_{max}=n^2a_o$ and for l=n-1 as it was mentioned before.

The probability density function for n=1 l=0 is shown in the next figure 3: $P(r) = r^2|R(r)|^2$

$\int_{r1}^{r2} P(r)dr = \int_{r1}^{r2} r^2\,|R(r)|^2 dr = \int_{x1}^{x2} 4x^2e^{-2x}dx$ where $x=r/a_o$ $dr=a_odx$

$P(x)=4x^2e^{-2x}$

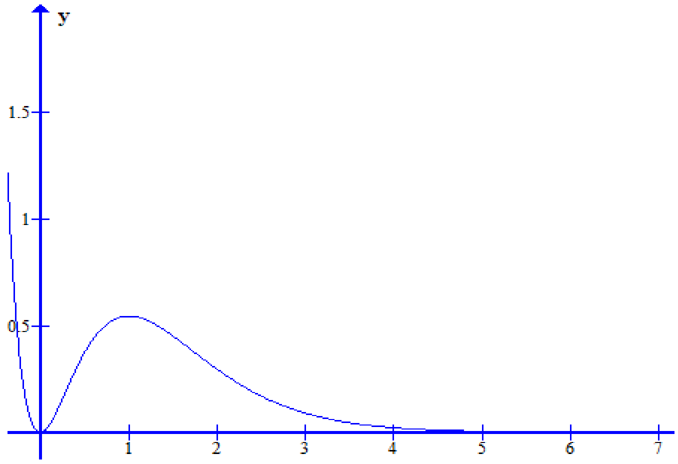

Fig.3. Probability density function P(x) versus $x=r/a_o$ for n=1 l=0

The probability density function for n=2 l=1 is shown in the next figure 4: $P(r) = r^2|R(r)|^2$

$\int_{r1}^{r2} P(r)dr = \int_{r1}^{r2} r^2\,|R(r)|^2 dr = \int_{x1}^{x2} \frac{x^4e^{-x}}{24}dx$ where $x=r/a_o$ $dr=a_odx$

$P(x)=\frac{x^4e^{-x}}{24}$

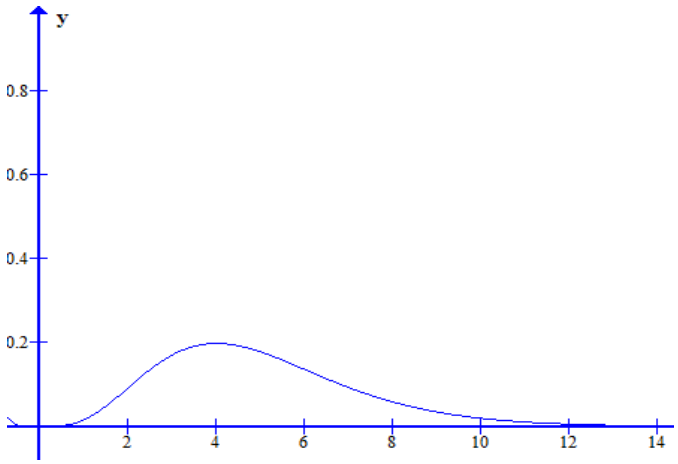

Fig.4. Probability density function P(x) versus $x=r/a_o$ for n=2 l=1

The probability density function for n=3 l=2 is shown in the next figure 5: $P(r) = r^2|R(r)|^2$

$\int_{r1}^{r2} P(r)dr = \int_{r1}^{r2} r^2\,|R(r)|^2 dr = \int_{x1}^{x2} \frac{16x^6e^{-(2x/3)}}{(27)^3*10}dx$ where $x=r/a_o$ $dr=a_odx$

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$P(x)=\frac{16x^6e^{-(2x/3)}}{(27)^3*10}$

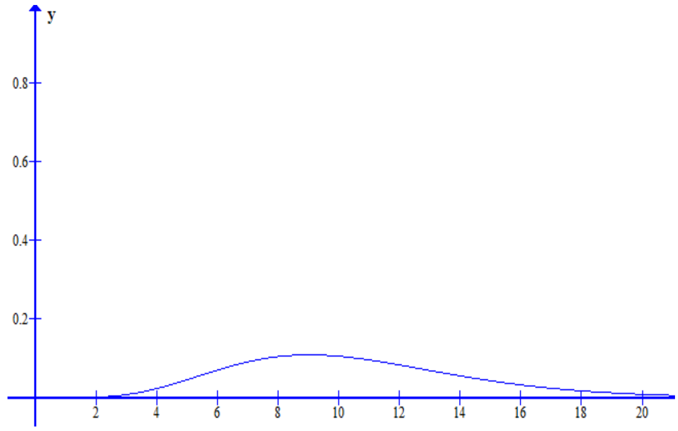

Fig.5. Probability density function P(x) versus $x=r/a_o$ for n=3 l=2

The probability density function for n=4 l=3 is shown in the next figure 6: $P(r) = r^2|R(r)|^2$

$\int_{r1}^{r2} P(r)dr = \int_{r1}^{r2} r^2\,|R(r)|^2dr = \int_{x1}^{x2} \frac{x^8e^{-x/2}}{322560*64}dx$ where $x=r/a_o$ $dr=a_o dx$

$P(x)= \frac{x^4e^{-x}}{24}$

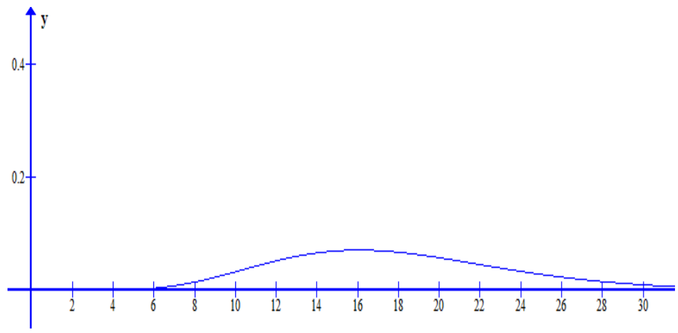

Fig.6. Probability density function P(x) versus $x=r/a_o$ for n=4 l=3

As an example, the calculation detail of the probability density of finding the electron between two distances r_1 and r_2 for the 1s state is given below. The radial wave function for 1s (n=1 l=0) is:

$$R(r) = \frac{2}{(a_o)^{3/2}}e^{-\frac{r}{a_o}}$$

$$\int_{r1}^{r2} P(r)dr = \int_{r1}^{r2} r^2\,|R(r)|^2dr$$

$$= \int_{r1}^{r2} r^2\,\frac{4}{{a_o}^3}\,e^{-\frac{2r}{a_o}}dr$$

For all radial wave functions of probability density, if $r_1=0$ and $r_2=\infty$, the answer is 1. For example, the probability density of finding the electron in the region $r_{max}-a_o \leq r \leq r_{max}+a_o$ ($r_{max}=n^2a_o$) $r_{max}=a_o$ corresponding to n=1 l=0 (s) (l=n-1) $0 \leq r \leq 2a_o$ is:

$r_o=a_o=0.529\,\dot{A}$ Bohr radius, radius for the first level of the Hydrogen atom

$$\int_0^{2ao} \mathrm{P(r)dr} = \int_0^{2ao} \mathrm{r}^2 \frac{4}{a_o{}^3}\, e^{-\frac{2r}{a_o}}\,\mathrm{dr}$$

$$= \frac{4}{a_o{}^3}\left(-\frac{a_o}{2}r^2 - \frac{a_o{}^2}{2}r - \frac{a_o{}^3}{4}\right)e^{-\frac{2r}{a_o}}\Big|_0^{2ao}$$

$=(1-13)*e^{-4}$

$=0.76$

The calculation detail of the probability density of finding the electron between two distances r_1 and r_2 for the 2p state is given below. The radial wave function for 2p (n=2 l=1) is:

$$\mathrm{R(r)} = \frac{1}{\sqrt{3}\,(2\mathrm{a_o})^{3/2}}\frac{\mathrm{r}}{\mathrm{a_o}}\,\mathrm{e}^{-\frac{\mathrm{r}}{2\mathrm{a_o}}}$$

$$\int_{\mathrm{r1}}^{\mathrm{r2}} \mathrm{P(r)dr} = \int_{\mathrm{r1}}^{\mathrm{r2}} \mathrm{r}^2\,|\mathrm{R(r)}|^2\mathrm{dr}$$

$$= \frac{1}{24a_o{}^5}\int_{\mathrm{r1}}^{\mathrm{r2}} r^4\,\mathrm{e}^{-\frac{\mathrm{r}}{\mathrm{a_o}}}\mathrm{dr}$$

For example, the probability density of finding the electron in the region $r_{max}-a_o \le r \le r_{max}+a_o$ ($r_{max}=n^2a_o$) $r_{max}=4a_o$ corresponding to n=2 l=1 (p) (l=n-1) 3 $a_o \le r \le 5a_o$ is:

$$\int_{3\mathrm{ao}}^{5\mathrm{ao}} \mathrm{P(r)dr} = \frac{1}{24a_o{}^5}\int_{3ao}^{5ao} r^4\,\mathrm{e}^{-\frac{\mathrm{r}}{\mathrm{a_o}}}\mathrm{dr}$$

$=0.375$

The calculation detail of the probability density of finding the electron between two distances r_1 and r_2 for the 3d state is given below. The radial wave function for 3d (n=3 l=2) is:

$$\mathrm{R(r)} = \frac{4}{27\sqrt{10}\,(3\mathrm{a_o})^{3/2}}\left(\frac{\mathrm{r}}{\mathrm{a_o}}\right)^2\mathrm{e}^{-\frac{\mathrm{r}}{3\mathrm{a_o}}}$$

$$\int_{\mathrm{r1}}^{\mathrm{r2}} \mathrm{P(r)dr} = \int_{\mathrm{r1}}^{\mathrm{r2}} \mathrm{r}^2\,|\mathrm{R(r)}|^2\mathrm{dr}$$

$$= \left(\frac{4}{27\sqrt{10}\,(3\mathrm{a_o})^{3/2}a_o{}^2}\right)^2\int_{\mathrm{r1}}^{\mathrm{r2}} r^6\,\mathrm{e}^{-\frac{2\mathrm{r}}{3\mathrm{a_o}}}\mathrm{dr}$$

For example, the probability density of finding the electron in the region $r_{max}-a_o \le r \le r_{max}+a_o$ ($r_{max}=n^2a_o$) $r_{max}=9a_o$ corresponding to n=3 l=2 (d) (l=n-1) $8a_o \le r \le 10a_o$ is:

$$\int_{8\mathrm{ao}}^{10\mathrm{ao}} \mathrm{P(r)dr} = \left(\frac{4}{27\sqrt{10}\,(3\mathrm{a_o})^{3/2}a_o{}^2}\right)^2\int_{8\mathrm{ao}}^{10\mathrm{ao}} r^6\,\mathrm{e}^{-\frac{2\mathrm{r}}{3\mathrm{a_o}}}\mathrm{dr}$$

$=0.13$

The calculation detail of the probability density of finding the electron between two distances r_1 and r_2 for the 4f state is given below. The radial wave function for 4f (n=4 l=3) is:

$$\mathrm{R(r)} = \frac{1}{\sqrt{322560}\,(\mathrm{a_o})^{3/2}}\left(\frac{\mathrm{r}}{2\mathrm{a_o}}\right)^3\mathrm{e}^{-\frac{\mathrm{r}}{4\mathrm{a_o}}}$$

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$$\int_{r1}^{r2} \mathrm{P(r)dr} = \int_{r1}^{r2} r^2\, |R(r)|^2 dr$$

$$= \left(\frac{1}{\sqrt{322560}\,(a_o)^{3/2}}\right)^2 \frac{1}{(2a_o)^6} \int_{r1}^{r2} r^8\, e^{-\frac{r}{2a_o}} dr$$

For example, the probability density of finding the electron in the region $r_{max}-a_o \leq r \leq r_{max}+a_o$ ($r_{max}=n^2a_o$) $r_{max}=16a_o$ corresponding to n=4 l=3 (f) (l=n-1) $15a_o \leq r \leq 17a_o$ is:

$$\int_{15ao}^{17ao} \mathrm{P(r)dr} = \left(\frac{1}{\sqrt{322560}\,(a_o)^{3/2}}\right)^2 \frac{1}{(2a_o)^6} \int_{15ao}^{17ao} r^8\, e^{-\frac{r}{2a_o}} dr$$

$$= 0.021$$

In the following table (3), values of r_{max} and P(r) are given for some states with principal quantum number n and orbital quantum number l=n-1 (momentum angular orbital quantum number):

Table 3. Value of the probability density of finding the electron in the shells between the two radius: $r_{max}-a_o \leq r \leq r_{max}+a_o$ where $r_{max}=n^2a_o$

State	rmax	value of the probability density of finding the electron in the shells between the two radius: rmax-ao≤r≤rmax+ao
1s	1ao	0.8
2p	4ao	0.375
3d	9ao	0.13
4f	16ao	0.021

Therefore, the variation of the probability density of the electron P(r) with respect to r for these shells in which the electrons are bound to protons is obtained from the solution of the Schrödinger equation. Besides, the mathematical solution of the Schrödinger equation for the Coulomb potential hydrogen atom depends on the quantum numbers n, l, m_l for different shells or energy levels [6].

In addition to the probability of finding electrons in the shells, these shells are related to the distribution of the proton mass cloud. Thus, the mass cloud of the proton interacts with the mass cloud of the electron resulting in the bonding of the two particles with loss of mass cloud and with no loss of electrical charge. Therefore, this is evidence of the existence of the mass cloud.

In this mass symmetry, the charge of the particle is concentrated in its mass nucleus with an uncharged mass cloud around its nucleus. This mass cloud is located in the respective orbitals given by the Schrödinger equation where the orbitals represent the possible locations or places where the particle location is determined probabilistically by the respective Schröndiger equation as it was calculated before. The existence of this mass symmetry is right because for example in the Hydrogen atom and in the electron transition from one shell to another shell, the electron and the proton lose mass in the interaction of the mass cloud (converted to photons or electromagnetic radiation) but do not lose electric charge as it was mentioned before.

4. Particle Bonds

If two particles are close to each other, the mass cloud of one of them interacts with the mass cloud of the other particle. In this interaction, the loss of the mass clouds of the two particles will be converted into electromagnetic

energy according to Einstein's equation: $E=mc^2$ and the variant mass formula discovered and developed by myself [1]. Furthermore, as mentioned in the postulates, the mass m and the mass cloud m* of the same particle have the same value: m*=m. Besides, the mass of a particle cannot interact with the mass cloud of the same particle, neither partially nor totally. However, the interaction occurs between the mass cloud of one particle and the mass cloud of another particle, either partially or totally.

There are two kinds of particle bonds:

(1) When one particle is not the antiparticle of the other particle with partial interaction. When one particle is not the antiparticle of the other particle, the interaction between the two particles results in a partial loss of their mass clouds and electromagnetic energy is radiated or emitted. Therefore, these two particles are joined together and one of these examples is the electron and the proton in the hydrogen atom.

(2) When one particle is the antiparticle of the other particle with full interaction. When one particle is the antiparticle of the other particle, the masses of the particles and antiparticles are equal and oppositely charged. In the interaction of these two particles, there is a total loss of mass (mass plus mass cloud) of both particles, which is converted to electromagnetic energy ($E=mc^2$).

Hydrogen Atom

Consider a free electron and proton, but not far from each other, there are two interactions that will occur successively:

(1) The attraction between the positive charge of the proton and the negative charge of the electron that causes the electron to move towards the proton.

(2) When the electron reaches the 1s state, the mass cloud of the proton interacts with the mass cloud of the electron. Thus, their masses will be reduced because the loss of mass cloud is converted to electromagnetic energy ($E=mc^2$), but these interactions have no effect on the electrical charge of the proton and the electron, which is concentrated in the nucleus of the particles [1],[6].

During this analysis, the angular momentum orbital quantum number (l) and the orbital magnetic quantum number (m_l) have not been considered.

The Hydrogen Atom has one electron orbiting the nucleus which has one proton. The electronic configuration of the Hydrogen Atom is: $1s^1$.

The positive charge of the proton (P+) is concentrated in its mass nucleus with an uncharged mass cloud around its nucleus. This mass cloud is located in the respective orbitals given by the Schrödinger equation where the orbitals represent the possible locations or places where the particle location is determined probabilistically by the respective Schröndiger equation. Furthermore, like the proton, this also occurs with the particles π^+, μ^+, e^+ for example.

For other hand, part of the mass of the electron (e^-) without electrical charge is distributed in the mass cloud around the mass nucleus that contains the negative charge. Thus, the negative charge of the electron is concentrated in its

mass nucleus with an uncharged mass cloud around its nucleus. In addition, like the electron, this also occurs with the particles π^-, μ^-, P^- for example.

The distribution of the mass cloud outside the proton occurs in such a way that, if a free electron enters in one of the shells or allowed quantum energy levels of the hydrogen atom, a part of the mass cloud of the proton interacts with a part of the electron mass cloud. Thus, the mass of the interacting cloud is converted into electromagnetic energy and the proton bonds with the electron. Therefore, the two particles join together due to this interaction and the electrostatic force between the two particles. Then, the electron and proton are bound together in the hydrogen atom by the mass cloud of the electron and proton with some mass cloud lost in the interaction and converted to electromagnetic energy or photons ($E=mc^2$). Hence, the two particles join together. In this form, it is right this mass symmetry and the existence of a mass cloud, since the electron and the proton in the interaction of the mass cloud lose mass but do not lose electric charge.

Besides, in the formation of the Hydrogen atom, the electron-proton system when approaching get a potential energy of V=- 27.2 eV (13.6 eV*2) but later when the electron bond occurs in the shell or quantum state n=1, energy of 13.6 eV is emitted as electromagnetic energy or photons and the remaining 13.6 eV remains as kinetic energy of the electron. Thus, the Hydrogen atom has 13.6 eV of additional energy/mass than the sum of the energy/mass of the proton plus the electron. In this form, it is needed 13.6 eV to ionize the Hydrogen atom and expel the electron from the atom. Therefore, the mass cloud of the proton keeps the electron in the 1s shell or ground state bound and the electron cannot leave this shell or the atom without receiving some energy from the outside of this atom which is 13.6 eV. If the electron is expelled, the mass/energy reduction of the proton and electron is 13.6/2 eV for each particle. Besides, when the bond of the electron occurs in the shell or principal quantum state n=1, energy of 13.6 eV is emitted as it was mentioned before [1].

Muonic Atom

The muonic atom consists of one proton and one muon which have a negative charge. The mass of the muon is 105.7 MeV/c^2 or 207 m_e, the mass of the proton is 938.272 MeV/c^2 or 1836 m_e, and the reduced mass of the muonic atom is [6]:

$$m_\mu = \frac{207me*1836me}{207me+1836me} = 186\ m_e$$

The most probable radius and binding energy for the muon can be calculated from the equations:

$$r = \frac{n^2h^2\varepsilon_o}{\pi m_o Ze^2}$$

$$r = \frac{n^2h^2\varepsilon_o}{\pi 186mZe^2}$$

$$a_o = \frac{\varepsilon_o h^2}{\pi me^2} \qquad r = \frac{n^2a_o}{186Z}$$ (donde m es la masa del electrón, Z=1 y n=1)

$$r=\frac{a_o}{186}$$ (a_o: Bohr radius=5.3 *10^{-11} m) $r=2.84*10^{-4}$ nm

$$E = \frac{-z^2e^4m_o}{8\varepsilon_o{}^2h^2n^2}$$

$E = \frac{-z^2 e^4 186m}{8\varepsilon_o{}^2 h^2 n^2}$ $E = \frac{-186(13.6)Z^2}{n^2}$ (Z=1 n=1) E(eV)=2533 eV

By applying the formula of E(eV) versus r(nm), it is obtained the same result: E(eV)=0.7194/r (nm) r=2.84*10^{-4} nm E=2533 eV

If the natural logarithm is used for smaller values of r and by applying the equation E(eV)=0.7194/r (nm) which gives the variation of energy (E eV) as a function of distance (r nm), the coordinates of the muonic atom coincide with the figure (1) of E versus r:

E(eV)=0.7194/r (nm) ln E(eV)=ln (1/r nm)+ln (0.7194)

ln E(eV)=ln (1/r nm)-0.3294

r=2.84*10^{-4} nm: ln (1/2.84*10^{-4} nm)=8.16

E=2533 eV ln (2533 eV)=7.84

For the muonic atom (as well as for the hydrogen atom), values of r and energy E can be obtained for the other shells or levels with different quantum number (n), for example, for n=2:

$r = \frac{n^2 a_o}{186Z}$ n=2 a_o: Bohr radius=5.3 *10^{-11} m Z=1

r=0.001136 nm

E=-186 (13.6) Z^2/n^2 (Z=1 n=2) E(eV)=633 eV

By applying the formula of E(eV) versus r(nm), it is obtained the same result: E(eV)=0.7194/r (nm) r=0.001136 nm E(eV)=633 eV

r=0.001136 nm: ln (1/0.001136 nm)=6.78

E=633 eV ln (633 eV)=6.45

The graph of ln (E eV) versus ln (1/r nm) is shown below:

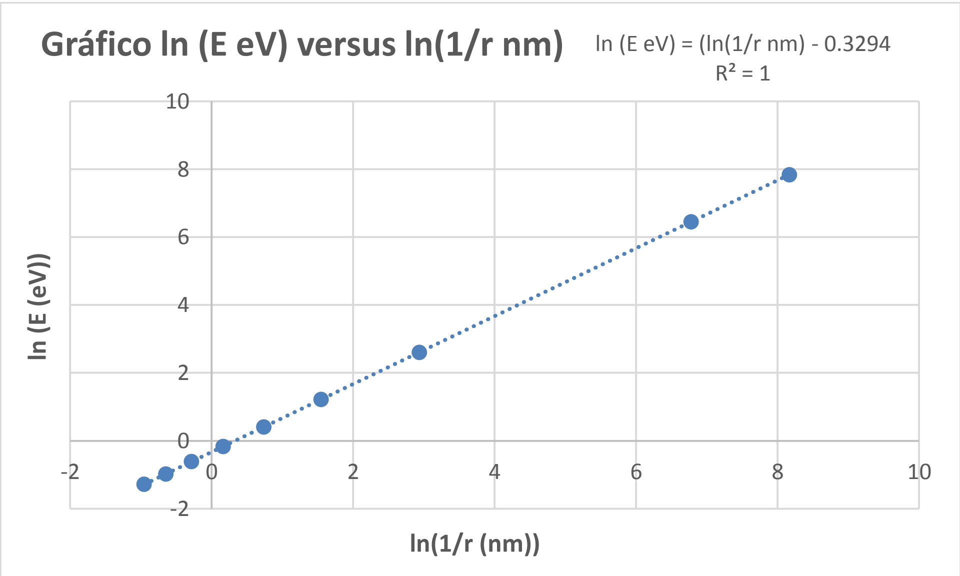

Fig.7. Variation of ln (EeV) versus ln(1/r(nm)) for the different shells or quantum levels n based on Bohr model

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The radial wave function for the muonic atom in the first shell is:

$$\mathrm{R(r)} = \frac{2}{{r_{max}}^{3/2}} e^{-\frac{r}{r_{max}}}$$

The probability density of finding the muon in the region between r=0 and r=2r_o corresponding to n=1 l=0 (s) (l=n-1) ($r_{max}=n^2r_o$ $r_{max}-r_o \le r \le r_{max}+r_o$)

$0 \le r \le 2r_o$ is:

$$\mathrm{R(r)} = \frac{2}{(r_o)^{3/2}} e^{-\frac{r}{r_o}} \qquad \mathrm{P(r)} = r^2|\mathrm{R(r)}|^2$$

r_o=2.84*10^{-4} nm

$$\int_0^{2ro} \mathrm{P(r)dr} = \int_0^{2ro} r^2\, |\mathrm{R(r)}|^2 \mathrm{dr}$$

$$\int_0^{2ro} \mathrm{P(r)dr} = \int_0^{2ro} r^2 \frac{4}{{r_o}^3}\, e^{-\frac{2r}{r_o}} \mathrm{dr}$$

$$= \frac{4}{{r_o}^3}\left(-\frac{r_o}{2}r^2 - \frac{{r_o}^2}{2}r - \frac{{r_o}^3}{4}\right) e^{-\frac{2r}{r_o}} |^{2ro}{}_{0}$$

$$=(1-13)*e^{-4}$$

$$=0.76$$

Inside the muonic atom and in the orbits of the atom is the muon (like the electron in the hydrogen atom). The muon has a negatively charged mass nucleus and an uncharged mass cloud surrounding the mass nucleus. Besides, the proton has a positively charged mass nucleus and an uncharged mass cloud surrounding the mass nucleus.

Thus, the mass cloud that interacts between the proton and the muon is converted into electromagnetic energy and the proton bonds with the muon. In this form, when the muon bonds (in the shell or main quantum state n=1) with the proton, 2533 eV of energy is emitted. The mass cloud reduction of the proton and the muon is 2533/2 eV for each particle during the formation of the muon atom.

Deuteron

There is the strong nuclear interaction between proton with neutron, neutron with neutron, and proton with proton. In addition to the strong interaction between two protons, there is also a weak Coulomb repulsion between the two protons. The Deuterio Atom has one electron orbiting the nucleus which has one proton and one neutron. The electronic configuration of the Deuterio Atom is: $1s^1$. The Deuterio is an isotope of the Hydrogen. The deuteron is the nucleus of deuterio which has two nucleons: a proton and a neutron and between which there is the strong interaction. It is possible to explain the strong interaction between these two nucleons by means of the effect of the mass cloud of one particle with the mass cloud of the other particle.

The binding energy of the deuteron is 2.23 MeV, which is the energy released when a proton and a neutron join to form a nucleus. Experiments show that the radius of the proton is about 1.2 fm and the radius of the neutron is about the same radius. For other hand, experiments show that the radius of the deuteron is about 2.21 fm. The

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distance between the centers of the two nucleons: proton and neutron is about 1 fm. Therefore, the proton and neutron in the deuteron are very close together and under this condition, the mass clouds overlap each other. Then, the mass cloud of the proton interacts with the mass cloud of the neutron and the mass of the interacting cloud is converted into electromagnetic energy or photons ($E=mc^2$) and the two nucleons bond [1],[6].

Then, as a result of the interaction between the mass clouds of the two nucleons, the two nucleons bind together tightly within this short range distance. But if by some external effect the distance between the proton and the neutron increases by a few fm, the strong force of interaction between the mass clouds will decrease, which is in fact part of the properties of the strong nuclear force: the nuclear force has a strong interaction at short range distances.

Ionized Helium Atom

The Helium Atom has two electrons orbiting the nucleus which has two protons and two neutrons. The electronic configuration of the Helium Atom is: $1s^2$. For other hand, if an electron is removed from the Helium atom, the Helium ion He^+ is formed. Helium ionized with one electron is like the hydrogen atom except Z=2. The electronic configuration of the Ionized Helium Atom is: $1s^1$.

Therefore, the most probable radius in the ground state with shell or quantum level 1s is:

$r = \frac{n^2 a_o}{Z}$ (a_o: Bohr radius=$5.3 * 10^{-11}$ m) n=1 Z=2

r=0.0265 nm

And the energy is: $E = \frac{-13.6 z^2}{n^2}$ n=1 Z=2

E=54.4 eV

$$(E)(r) = \left(\frac{-z^2 e^4 m_o}{8\varepsilon_o{}^2 h^2 n^2}\right) \left(\frac{n^2 h^2 \varepsilon_o}{\pi m_o Z e^2}\right)$$

$$= \frac{-Ze^2}{8\pi\varepsilon_o} \quad Z=2$$

(E)(r)=2*0.7194 eV-nm

E(eV)=2*0.7194/r (nm) r=0.0265 nm E=54.4 eV

In this form, the only two protons in this nucleus affect these values: r_{max} and E, while the neutrons have no effect on the electron [6]. Since the mass cloud of the two protons of the helium ion compared to that of the hydrogen atom which has one proton is larger, the electron in this shell or quantum level 1s is more strongly bound: 54.4 eV (He^+)>13.6 eV (H).

Helium Nucleons

The Helium nucleus or α particle with a radius of r=1.9 fm has two protons and two neutrons [1],[6]. In the interaction between the mass clouds of these nucleons, the energy emitted ($E=mc^2$) in the bond of the Helium nucleons is 28.3 MeV.

Nucleons

Consider a heavy nucleus that has many nucleons: protons and neutrons. The mass clouds of nucleons within the nucleus interact with each other without any effect on the proton charge. For a heavy nucleus, the average value of the binding energy of each nucleon is about 8 MeV.

5. Particle bonds between Particle and Antiparticle

When one particle is the antiparticle of the other particle, the masses of the particles and antiparticles are equal and oppositely charged. In the interaction of these two particles, there is a total loss of mass (mass plus mass cloud) of both particles, which is converted to electromagnetic energy ($E=mc^2$).

A particle and an antiparticle are the same particles in terms of their properties including their masses except for their electrical charges. Thus, if the particles have an electrical charge, one particle has a positive charge and the other particle has a negative charge. The most frequently found particle in nature is called a particle. However, for a pair of particles that do not have an electrical charge, one particle is arbitrarily called a particle and the other particle an antiparticle [1],[6].

Besides, if a particle and its antiparticle are close to each other, the mass cloud of one of the particles interacts with the mass cloud of the other particle. In this interaction, the total mass of the particle m and the antiparticle m* (m*=m for particle and antiparticle) is converted into electromagnetic radiation emitted in the form of electromagnetic rays or photons.

Electron and Positron

The electron has a negative electrical charge with mass m and its antiparticle called positron has a positive electrical charge with mass m* where m=m*. In the interaction of the electron with the positron, the total mass (mass plus mass cloud) of the electron and positron is converted into electromagnetic energy and two gamma rays or photons with energy of 0.511 MeV each one are produced. Two photons are needed to hold conservation of momentum [1],[6].

$e^- + e^+ \rightarrow \gamma_1 + \gamma_2$

$E_{\gamma 1} + E_{\gamma 2} = 2mc^2$ where m is the electron mass and $m=m^*$ $m=0.511$ MeV/c^2

Proton and Antiproton

The interaction of the proton (P^+) with the antiproton (P^-) can be achieved experimentally in accelerators or particle colliders installed as for example at CERN, DESY, SLAC, Fermilab [1],[6]. If the proton and antiproton interact, the total mass (mass plus mass cloud) of the proton and antiproton is converted into electromagnetic energy and two gamma rays or photons with energy of 938.28 MeV each one are produced. Two photons are needed to hold conservation of momentum.

$P^- + P^+ \rightarrow \gamma_1 + \gamma_2$

$E_{\gamma 1} + E_{\gamma 2} = 2mc^2$ where m is the proton mass and $m=m^*$ $m=938.28$ MeV/c^2

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Muon and antimuon

The masses of the muon and the antimuon are equal. If the muon and antimuon interact, the total mass (mass plus mass cloud) of the muon and antimuon is converted into electromagnetic energy and two gamma rays or photons with energy of 105.6 MeV each one are produced [1],[6]. Two photons are needed to hold conservation of momentum.

$\mu^{-}+\mu^{+} \rightarrow \gamma_1+\gamma_2$

$E_{\gamma 1}+E_{\gamma 2}=2mc^2$ where m is the muon mass and $m=m^*$ $m=105.6$ MeV/c^2

Neutral Pion and neutral antipion

The masses of the neutral pion and the neutral antipion are equal and without electrical charge. If the neutral pion and neutral antipion interact, the total mass (mass plus mass cloud) of the neutral pion and neutral antipion is converted into electromagnetic energy and two gamma rays or photons with energy of 134.97 MeV each one are produced. Two photons are needed to hold conservation of momentum. Between these two particles: neutral pion and neutral antipion, the selection of particle and antiparticle is arbitrary [1], [6].

$\pi^{o}+\bar{\pi}^{o} \rightarrow \gamma_1+\gamma_2$

$E_{\gamma 1}+E_{\gamma 2}=2mc^2$ where m is the neutral pion mass $m=m^*$ $m=134.97$ MeV/c^2

6. Bound of Diatomic Molecules

Bound of Diatomic Molecules

It is possible to do the analysis for the center of mass (CM) as follows:

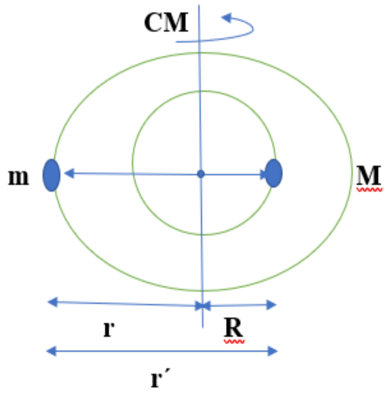

Fig.8. Center of mass for diatomic molecules

$mr=MR$ $R=(m/M)r$ $r'=r+R$ $r'= r+(m/M)r$

$r'=\frac{M+m}{M}r$ $r=\frac{M}{M+m}r'$

$m\frac{v^2}{r}=\frac{kZe^2}{r'^2}$

$v^2=r\frac{kZe^2}{mr'^2}=\frac{M}{M+m}r'\frac{kZe^2}{mr'^2}=\frac{kZe^2}{2m_o r'}$ $M=m=m_o$

$v = \sqrt{k\frac{Ze^2}{2m_o r'}}$

The reduced mass of the Diatomic Molecule is:

$m_{RDM} = \frac{mM}{m+M}$

Hydrogen Molecule H_2

Firstly, we consider the Hydrogen molecule H_2 which is formed due the bond of two Hydrogen Atoms H where each Hydrogen Atom H consists of one proton and one electron. The electronic configuration of the Hydrogen Atom $1s^1$. Nowadays, this bond of the Hydrogen molecule H_2 is explained by mean of the covalent bond. In the hydrogen molecule H_2, the electrons in the shell 1s circulate between the two hydrogen atoms. Therefore, the two electrons are shared by the two Hydrogen Atoms H. The two electrons can be shared if the spins are in opposite direction. Besides, the electrons of both atoms remain in the zone between them longer than in any other zone, which produces an attractive force of the protons towards this zone which explains the covalent bond. These types of forces are the forces that form homonuclear diatomic molecules, such as the Hydrogen molecule, in which for this reason both electrons remain attached to the two protons of both atoms [1],[4],[5],[7].

Nevertheless, this covalent bond can be explained by means of the mass symmetry: mass and cloud mass. If the two atoms of Hydrogen H are close to each other, the mass cloud of the proton of one atom of H interacts with the mass cloud of the electron of the same atom and with the mass cloud of the electron of the other atom because the electrons are shared by the two Hydrogen atoms. The same occurs for the proton of the other Hydrogen atom. In this interaction, the loss of the mass clouds that interacts will be converted into electromagnetic energy according to Einstein's equation: $E=mc^2$ and the variant mass formula discovered by myself [1].

The bond energy that corresponds to the electromagnetic energy emitted when the bond occurs is calculated with the variant mass formula developed by myself as follows:

The mass of Hydrogen Atom H is: $m_o=1.673534 * 10^{-27}$ kg.

The reduced mass of the Hydrogen Molecule is:

$m_{RH2} = \frac{m_o\, m_o}{m_o+m_o} = \frac{m_o}{2}$ where m_o is the mass of the Hydrogen Atom

The internuclear distance is: $r'=0.74$ A (1 A=10^{-10} m).

$v = \sqrt{k\frac{Ze^2}{2m_o r'}}$ $\quad k=\frac{1}{4\pi\epsilon_o}=9*10^9\ Nm^2/C^2 \quad Z=1 \quad e=1.602*10^{-19}$ C

v=30537.65179 m/s

$m = m_o e^{-(\frac{v^2}{2c^2})}$ where c is the light velocity $c=3*10^8$ m/s (v_o=0 m/s)

$\Delta mc^2 = m_o c^2 - (m_o e^{-(\frac{v^2}{2c^2})})c^2$

Δmc^2=4.87 eV

If it is used the formula of E versus r (Bohr approach), it is obtained the same results:

$$(E)(r)= \left(\frac{-z^2e^4m_o}{8{\varepsilon_o}^2h^2n^2}\right)\ \left(\frac{n^2h^2\varepsilon_o}{\pi m_o Ze^2}\right)$$

$$=\frac{-Ze^2}{8\pi\varepsilon_o} \qquad Z=1$$

(E)(r)=0.7194 eV-nm

E(eV)=0.7194/r (nm)

E(eV)=0.7194/r(nm) where r is the double of the internuclear distance:

r=2r´

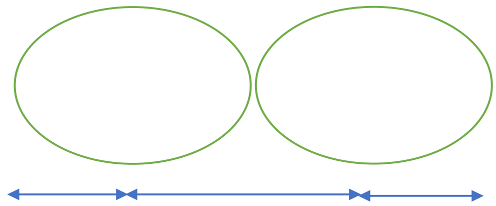

Fig.9. Diatomic Molecule and the maximum distance for the shared electron: 2r´ (Hydrogen Molecule)

The distance r is: 2*0.074 nm

E(eV)=0.7194/(2*0.074) eV

E=4.86 eV

The experimental value for the bond energy for the Hydrogen molecule H_2 is 4.72 eV. This bond energy of the Hydrogen molecule H_2 is lower than the bond energy between the electron and the proton in the Hydrogen Atom: 4.72 eV<13.6 eV. It is because of the electrostatic force of repulsion of the two shared electrons in the Hydrogen molecule H_2. Also, the internuclear distance between the two Hydrogen Atoms (0.74 A) is greater than the distance between the electron and the proton in the Hydrogen Atom (0.53 A) in the shell 1s: 0.74 A>0.53 A [1],[4],[5],[7].

Ionized Hydrogen Molecule H_2^+

The Hydrogen molecule H_2 is formed by the covalent bond of two Hydrogen Atoms where each Hydrogen Atom H consists of one proton and one electron. The electronic configuration of the Hydrogen Atom $1s^1$. Thus, the two electrons of the Hydrogen molecule H_2 are shared by the two Hydrogen Atoms H. The two electrons can be shared if the spins are in opposite direction [1],[4],[5],[7].

For other hand, the Ionized Hydrogen molecule H_2^+ is formed due the bond of two Hydrogen Atoms H but with one electron expelled from the H_2 molecule. Nowadays, this bond of the Ionized Hydrogen molecule H_2^+ is explained by mean of the covalent bond. In the Ionized hydrogen molecule H_2^+, the electron in the shell 1s circulate between the two hydrogen atoms. Therefore, the electron is shared by the two Hydrogen Atoms H. Besides, the electron of both Hydrogen atoms remain in the zone between them longer than in any other zone, which produces

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an attractive force of the protons towards this zone which explains the covalent bond. Therefore, the electron remains attached to the two protons of both atoms due this reason [1],[4],[5],[7].

Nevertheless, this covalent bond can be explained by means of the mass symmetry: mass and cloud mass. If the two atoms of Hydrogen H are close to each other, the mass cloud of the proton of one atom of H interacts with the mass cloud of the electron shared by the two Hydrogen atoms. The same occurs for the proton of the other Hydrogen atom. In this interaction, the loss of the mass clouds that interacts will be converted into electromagnetic energy according to Einstein's equation: $E=mc^2$ and the variant mass formula discovered by myself [1]. The bond energy that corresponds to the electromagnetic energy emitted when the bond occurs is calculated with the variant mass formula developed by myself as follows:

The mass of Hydrogen Atom H is: $m_o=1.67353 * 10^{-27}$ kg.

The internuclear distance is: $r´=1.06$ A (1 A=10^{-10} m).

$$v = \sqrt{k\frac{Ze^2}{2m_o r´}} \quad k=\frac{1}{4\pi\epsilon_o}=9*10^9\ Nm^2/C^2 \quad Z=1\ e=1.6*10^{-19}\ C$$

v=25515.17586 m/s

$$m = m_o e^{-(\frac{v^2}{2c^2})} \quad \text{where c is the light velocity } c=3*10^8 \text{ m/s}$$

$$\Delta mc^2= m_o c^2-(m_o e^{-(\frac{v^2}{2c^2})})c^2$$

$\Delta mc^2=3.40$ eV

If it is used the formula of E versus r (Bohr approach), it is obtained the same results:

$$(E)(r)= (\frac{-z^2e^4m_o}{8\varepsilon_o{}^2h^2n^2})\ (\frac{n^2h^2\varepsilon_o}{\pi m_o Ze^2})$$

$$=\frac{-Ze^2}{8\pi\varepsilon_o} \qquad Z=1$$

(E)(r)=0.7194 eV-nm

E(eV)=0.7194/r (nm)

E(eV)=0.7194/r(nm) where r is the double of the internuclear distance:

r=2r´

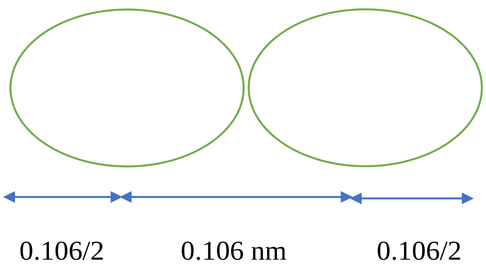

Fig.10. Diatomic Molecule and the maximum distance for the shared electron: 2r´ (Ionized Hydrogen Molecule)

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The distance r is: 2*0.106 nm

E(eV)=0.7194/(2*0.106) eV

E=3.39 eV.

The experimental value for the bond energy for the Ionized Hydrogen molecule H_2^+ is 2.65 eV. The bound energy for H_2 is not the double of the bound energy for H_2^+, because the repulsion between the electrons of the H_2 which decrease the bound from 5.3 eV to 4.72 eV and the distance is 0.74 A instead of 0.53 A which is the internuclear distance of H_2^+ divided by 2: 1.06 A / 2 [1],[4],[5],[7].

Therefore, the molecule of H_2 is more stable than the molecule of ionized hydrogen H_2^+. The bound energy for the H_2^+ is less intense than for H_2: 2.65 eV<4.72 eV. Besides, the internuclear distance of the Ionized Hydrogen Molecule H_2^+ (1.06 A) is greater than the internuclear distance of the Hydrogen Molecule H_2 (0.74 A): 1.06 A>0.74 A.

Oxygen Molecule O_2

The electronic configuration of the oxygen atom O is: $1s^2$ $2s^2$ $2p^4$. The oxygen atom consists of 2 electrons in the first level n=1 and 6 electrons in the second level n=2. The Oxygen molecule O_2 is formed by the covalent bond of two Oxygen Atoms O where each Oxygen Atom O consists of eight protons, eight neutrons and eight electron: 2 electrons in first level and 6 electrons in second level. Besides, two electrons of the second level are shared by each oxygen atom O. Then, there are four electrons shared totally to form the covalent bond [1],[4],[5],[7].

Nowadays, this bond of the Oxygen Molecule O_2 is explained by mean of the covalent bond. In the Oxygen Molecule, four electrons of the second level circulate between both oxygen atoms. Besides, the four shared electrons remain in the zone between them longer than in any other zone, which produces an attractive force of the protons towards this zone which explains the covalent bond. Therefore, the four electrons remains attached to the protons of both atoms due this reason [1],[4],[5],[7].

Nevertheless, this covalent bond can be explained by means of the mass symmetry: mass and cloud mass. If the two atoms of Oxygen O are close to each other, the mass cloud of the protons of one atom of O interacts with the mass cloud of the electrons shared by the two Oxygen atoms. The same occurs for the protons of the other Oxygen atom. In this interaction, the loss of the mass clouds that interacts will be converted into electromagnetic energy according to Einstein's equation: $E=mc^2$ and the variant mass formula discovered by myself [1].

The bond energy that corresponds to the electromagnetic energy emitted when the bond occurs is calculated with the variant mass formula developed by myself as follows:

The mass of Oxygen Atom O is: $m_o=2.65*10^{-26}$ kg.

The internuclear distance is: r=1.21 A (1 A=10^{-10} m).

$v=\sqrt{k\frac{Ze^2}{2m_o r}}$ $k=\frac{1}{4\pi\epsilon_o}=9*10^9\ Nm^2/C^2$ Z=2 $e=1.6*10^{-19}$ C

v=8487.27 m/s

$m = m_o e^{-(\frac{v^2}{2c^2})}$ where c is the light velocity c=3*10^8 m/s

$\Delta mc^2 = m_o c^2 - (m_o e^{-(\frac{v^2}{2c^2})})c^2$

Δmc^2=5.96 eV

If it is used the formula of E versus r (Bohr approach), it is obtained the same results:

$(E)(r) = (\frac{-z^2 e^4 m_o}{8\varepsilon_o{}^2 h^2 n^2})\ (\frac{n^2 h^2 \varepsilon_o}{\pi m_o Z e^2})$

$= \frac{-Ze^2}{8\pi\varepsilon_o}$ Z=2

(E)(r)=2*0.7194 eV-nm

E(eV)=(2*0.7194)/r (nm)

E(eV)=(2*0.7194)/r(nm) where r is the double of the internuclear distance:

r=2r´

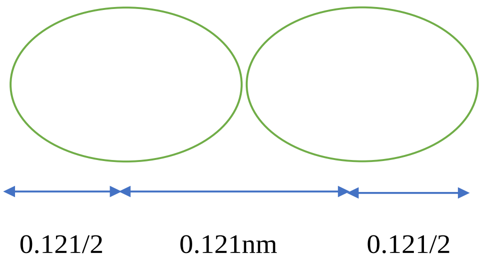

Fig.11. Diatomic Molecule and the maximum distance for the shared electron: 2r´ (Oxygen Molecule)

The distance r is: 2*0.121 nm

E(eV)=(2*0.7194)/(2*0.121) eV

E=5.95 eV

La energía de enlace experimental del O_2 es de 5.08 eV

It is possible to calculate the rotation frequency w for the O_2:

$L = Iw \approx \frac{h}{2\pi}$

$w \approx \frac{h}{2\pi I}$

The mass of Oxygen Atom O is: m_o=2.65 *10^{-26} kg.

The internuclear distance is: r=1.21 A (1 A=10^{-10} m).

$I = 2m_o(r/2)^2 = 1.94*10^{-46}$ kg m^2

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By replacing the value of the Planck constant h, it is obtained:

$w=5.43*10^{11}$ Rad/s

It is in accordance with the experimental measured for the rotation frequency [1]. The experimental value of the angular velocity of the oxygen molecule is in the order of 10^{11} Rad/s. The rotation frequency is lower than the vibration frequency which is in the order of 10^{13} Hz .

The kinetic energy of rotation of the oxygen molecule is:

$K = \frac{1}{2}Iw^2 \quad I=1.94*10^{-46} \quad w=5.43*10^{11}$ Rad/s

$K=2.86*10^{-23}$ J $\qquad K=0.000179$ eV

The kinetic energy of translation of the oxygen molecule is of the order of $6*10^{-21}$ J.

7. Conclusions

Since in nature there is symmetry, there should also be symmetry in physics since physics describes the phenomena of nature. In Physics, there is symmetry between the forces and their interactions: electric charges, magnetic poles, spins. Then, the particle mass must also have this symmetry: mass duality. For convenience and due to later explanations, I call this mass symmetry or mass duality as follows: mass and mass cloud.

If two particles are close to each other, the mass cloud of one of them interacts with the mass cloud of the other particle. In this interaction, the loss of the mass clouds of the two particles will be converted into electromagnetic energy according to Einstein's equation: $E=mc^2$ and the variant mass formula discovered and developed by myself. Furthermore, as mentioned in the postulates, the mass m and the mass cloud m* of the same particle have the same value: m*=m. Besides, the mass of a particle cannot interact with the mass cloud of the same particle, neither partially nor totally. However, the interaction occurs between the mass cloud of one particle and the mass cloud of another particle, either partially or totally. If the interaction is between particle and antiparticle, there is a conversion of the total mass (mass and cloud mass) in electromagnetic energy or photons.

For the proton, part of the mass of the uncharged proton is distributed in the orbital or mass cloud around the mass that contains the positive charge. Thus, the positive charge in the proton is concentrated in its mass nucleus with an uncharged mass cloud around its nucleus distributed in the orbitals. For the electron, part of the mass of the uncharged electron is distributed in the orbital or mass cloud around the mass that contains the negative charge. Thus, the negative charge in the electron is concentrated in its mass nucleus with an uncharged mass cloud around its nucleus distributed in the orbitals.

For example, in the formation of the hydrogen atom, a part of the mass cloud of the proton interacts with the mass cloud of the electron, and the total mass energy lost in this interaction is transformed into electromagnetic energy according to Einstein's equation: $E=mc^2$ and the variant mass formula discovered and developed by myself. Then, the two particles join together due to this interaction and the electrostatic force between the two particles. Therefore, the electron and proton are bound together in the hydrogen atom by the mass cloud of the electron and

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proton with some mass cloud lost in the interaction and converted to electromagnetic energy or photons. Then, it is right this mass symmetry, since the electron and the proton in the interaction of the mass cloud lose mass but do not lose electric charge. In this form, it is justified the existence of a mass cloud.

In the formation of the Hydrogen atom, the electron-proton system when approaching gains a potential energy of 27.2 eV (13.6 eV*2) but then when the electron bond occurs in the shell with quantum state n =1, energy of 13.6 eV is emitted as electromagnetic energy or photons and the remaining 13.6 eV remains as kinetic energy of the electron. Then, the Hydrogen atom has 13.6 eV of additional energy/mass than the sum of the energy/mass of the proton plus the electron. Therefore, 13.6 eV is needed to ionize the Hydrogen atom and expel the electron from the atom. The mass/energy reduction of the proton and electron is 13.6/2 eV for each particle due the emission of 13.6 eV as electromagnetic energy.

The mass symmetry can be corroborated in the experiments of the hydrogen spectrum, the Bohr model and the solution of the Schrödinger equation. In this mass symmetry, the mass cloud is located in the respective orbitals given by the Schrödinger equation. The orbitals represent the possible locations or places of the particle which is determined probabilistically by the respective Schröndiger equation.

The mathematical solution of the Schrödinger equation for the hydrogen atom with Coulomb potential energy, is obtained for the quantum numbers n, l, ml for different quantum energy levels or energy shells. The figures (3),(4), (5),(6) show the variation of the electron probability density P(r) versus r for these shells, in which the electron and proton are bound together in the hydrogen atom.

In addition to the probability of finding electrons in these shells, the bond is produced by the distribution of the mass cloud of the proton. Thus, the mass cloud of the proton interacts with the mass cloud of the electron, resulting in the bonding of the two particles.

Also, the mass symmetry is demonstrated for the Ionized Helium Atom, Helium Nucleus and the Muonic Atom. The extrapolation of the energy (figures 1 and 7) of the shells or energy levels (E) versus (r) for the hydrogen atom is used for the muonic atom where the muon is in the shell or energy level instead of the electron. The interaction of the mass cloud of the muon and the proton produces the bond of the two particles. The coordinates of r and E coincide with the extrapolation of the curve of E versus r.

In the interaction between the particle and its antiparticle, the conversion of energies of the total mass (mass and mass cloud) of the particle and antiparticle into electromagnetic energy takes place. The mass symmetry is demonstrated for the Proton and Antiproton, Muon and Antimuon, Neutral Pion and Neutral Antipion.

The main function of the mass cloud is the binding energy. The mass cloud interaction generates binding energy between the electrons and the nucleus in the atom through the protons and between the nucleons in the nucleus: protons with protons, neutrons with neutrons, and protons with neutrons. The nuclear force between two nucleons is characterized by being strong and short-range. These properties of the nuclear force between nucleons are explained by the interactions of the mass clouds between the nucleons. In addition, the mass symmetry is demonstrated for Diatomic Molecules as the Hydrogen molecule H_2, Ionized Hydrogen molecule H_2^+ and the

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Oxygen Molecule O_2. The results of the application of the variant mass formula in obtaining the binding energy largely agree with the experimental results.

Therefore, this scientific research presents evidence of the existence of the mass symmetry based in the Einstein's equation and in the Variant Mass formula for the Electron in the atom discovered and demonstrated by myself where theoretical and experimental results are detailed.

Declarations

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Consent for publication

Author declares that he/she consented for the publication of this research work.

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